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THE SPECTRAL BALANCE : A GENERAL METHOD FOR ANALYSIS OF NONLINEAR MICROWAVE CIRCUITS DRIVEN BY NON-HARMONICALLY RELATED GENERATORS

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INTRODUCTION

A very wide class of non-linear microwave circuits including mixers, oscillators (with their noise generators) and multitone amplifiers, are driven by non-harmonically related signals, and involve very far or closely-spaced frequency components.

In this paper an efficient method is proposed to perform the balance of non-linear circuits driven by non-harmonically related input signals. It may be named : The spectral balance.

I - THE NEW DISCRETE FOURIER TRANSFORM

General form : arbitrary signal x(t)

An arbitrary signal x(t) periodic or not, with a discrete finite spectrum of M+1 positive frequencies, may be expressed as a generalized Fourier series :

$$x(t) = x_o + \sum_{k=1}^{M} x_k \exp(j2\Pi f_k t) + x_{-k}\exp(-j2\Pi f_k t) \qquad (1)$$

with $f_{k+1} > f_k$ and $(f_{k+1}-f_k) > \Delta f$, k=0,1...M-1

The different frequencies f_k have no particular relation ship : Δf is the minimum separation between two adjacent frequencies.

If we multiply x(t) by $\exp(-j2\Pi f_k t)$ we translate left its spectrum by an amount f_k : if then we pass that signal through a low pass filter of bandwith Δf, we get at output a complex dc signal of amplitude equal to the coefficient x_k of the generalized Fourier series (1). By varying the translating frequency f_k, we obtain all the spectrum :

$f_k = m1.f1+m2.f2+...+mp.fp$ where $f_1,...,f_p$ are input frequencies.

The main problem in this method is to design the very narrow low pass filter. But digital filters design techniques are now well known [4]. In our application it is necessary that none of the frequencies of the spectrum falls inside a pass band other than that around f=0, for all the translations. The most general condition for this to be possible is to choose the sampling frequency f_s such that :

$$f_s > 2.f_M + \Delta f \qquad (2)$$

where f_M is the highest frequency of the spectrum.

Pseudo modulated signals

In a large number of microwave applications, signals do not have dense spectra, but instead spectra such that frequencies with not negligeable energy are concentrated around some pseudo carriers mf_0 (mf_0 may not be part of the spectrum) m=0,1,2...L.

For such a spectrum, following the same principle as described in the previous section, it is not necessary to choose f_s as imposed by the extreme condition (2), but rather one can choose a sampling frequency very much less than the highest frequency f_M of the spectrum, provided it locates the pass bands of the filter such that only the lowpass passband will pick energy from the spectrum translated signal.

It is easily proved that the sampling frequency f_s satisfying the filtering criterion is such that [Fig.1] :

$$\begin{cases} f_s > (2L+1)B \\ P.f_s < f_0 - B \\ (2L.P+1)f_s > 2L.f_0 + B \end{cases} \qquad (3)$$

where : P is the highest integer satisfying (3).
L the number of pseudo carriers (or of such subspectra).
B is the maximum width of the subspectra.

Since the sampling frequency given by (3) is in general very much less than the highest frequency f_M, the number of signal samples is also very much lower than that would be necessary with condition (2) of the general case, and for the classical DFT, if the signal were periodic. It is proved from (3) that the gain on the number samples, over the classical DFT (when the signal is periodic) is of order :

$$G = 2.L.P+1 \qquad (4)$$

As P varie from 0 to more than 10^9 in microwaves, we then see clearly the interest of this new approach.

Let us now explain the second key improvement allowing to perform a truly spectral-balance without prohibitive time consuming.

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II - CALCULATION OF THE EXPLICIT JACOBIAN NECESSARY IN THE NEWTON RAPHSON PROCESS

The equation describing a non-linear circuit in the frequency domain may be expressed [5] :

$$[F(X)] = [X] - [A_y | A_G] \begin{bmatrix} Y \\ G \end{bmatrix} = 0 \qquad (5)$$

Y, X and G are respectively vectors of generalized Fourier series coefficients (see section I) of :
- the non-linearities
- the non-linear controlling variables
- the external generators

A_Y and A_G are got by a classical analysis of the linear subnetwork.

The numerical resolution of the equation (5) is performed using the Newton Raphson process which needs to calculate the Jacobian [J] of F(X). Since X are the Fourier series coefficients of real functions, F is a function of X and X* (complex conjuguate) [6]. Then F is not an analytical function of the complex variable X. To circumvent this problem, each complex component is written as :

$$X = X^R + jX^I, X^R = \mathrm{Real}(X) \quad X^I = \mathrm{Imag}(X) \qquad (6)$$

The new system to solve is the following :

$$[F(X^R,X^I)] + [\frac{\partial F}{\partial X^R}] [\Delta X^R] + [\frac{\partial F}{\partial X^I}] [\Delta X^I] = 0 \qquad (7)$$

$\frac{\partial F}{\partial X^R}$ and $\frac{\partial F}{\partial X^I}$ have analytical expressions, then we used Newton Raphson process with an explicit Jacobian [J].

III - PRACTICAL EXAMPLES

III - 1 One Diode mixer

The first example which deals with one diode mixer is just given to prove that our Fourier Transform work well with the spectral balance technique.

The mixer circuit is shown in Fig.2. The input voltage of the mixer e(t) is composed of two frequencies f_1 and f_2 very close.

$$e(t) = A_1 \cos(2 \Pi f_1 t) + A_2 \cos(2 \Pi f_2 t)$$

A filter is placed accross the diode which shortens all the currents at frequencies other than the input frequencies f_1 and f_2. So the voltage accross the diode is composed only of the two frequencies f_1 and f_2, hence it is relatively easy to find the analytic expression of the Fourier expansion of the current using Bessel functions of first kind. The spectrum of i(t) is given by fig.3. The error is less than 0.2 % for all the component less than 90 dB below the maximum level, and only about 2 % components 110 dB below. We have condidered for the analysis 57 frequencies, the number of samples necessary was 420 and the program converged after 57 iterations with the error $\sum_k |F_k|^2$ less than 10^{-8}(k 1,... Nfreq).

Fig.3 is not on scale since $(f_1-f_2)/f_2$ is less than 10^{-5}

III - 2 Microwave mixer

The second example is a balanced mixer (Fig.5) with a microwave distributed circuit. The diode is represented by the equivalent circuit in Fig.4.

We have considered for the analysis 21 frequencies, the number of samples necessary was 212 and the program converged after 53 iterations, with the error $\sum_k |F_k|^2$ less than 10^{-8}(k 1,... Nfreq).

The spectrum of $v_d(t)$ is shown in Fig.6, but not on scale since $(f_1-f_2)/f_2$ is less than 10^{-3}.

CONCLUSION

An efficient and systematic technique for spectra calculation of both periodic and aperiodic signals have been developped.

Two key-features allow to perform a truly spectral balance without prohibitive time consuming :

- the first is a new discret Fourier Transform which allows to compute very quickly the spectra of periodic and aperiodic signals : for example the steady state response of a mixer, driven by two signals at frequencies f_1 and f_2, with a frequency ratio: $\frac{|f_1-f_2|}{f_1+f_2}$ as low as 10^{-9}, and a level ratio in the range of 0dB - 90 dB may be obtained with a good precision and computation cost (the noise of the D.F.T. used is about -130 dB below the mean value of all the spectral coefficients).

- the second key improvement is due to the explicit calculation in the frequency domain of the Jacobian in the Newton Raphson minimization process which reduces the number of iterations.

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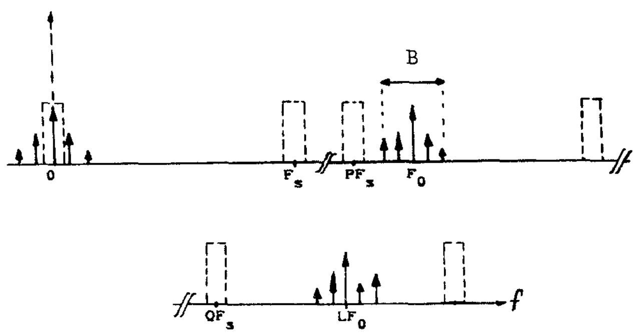

Fig.1 : Filtering figure of a pseudo modulated signal
The dashed curve represents the filter characteristic

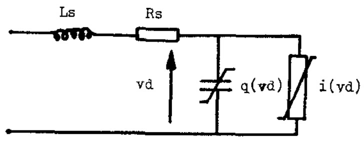

Fig.4 : Equivalent circuit of the diode
$Rs = 3.1\Omega$ $Ls = 1$ nH
$id(vd) = 1E\text{-}9(\exp(34.8 x vd(t) - 1)A$
$q(vd) = -.9E\text{-}12(1 - vd(t)/.61)^{1/2}Cb$

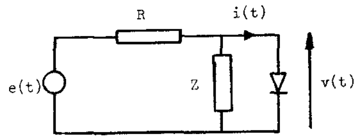

Fig.2 : Diode example
$i(t) = 10E\text{-}9\ (\exp(34.8 x v(t)) - 1)A$
$R = 50\Omega$, $Z(f) = \infty$ f=f1 and f=f2
0 otherwise
$e(t) = 1V\cos(2\Pi f_1 t) + 0.2V\cos(2\Pi f_2 t)$
$f_1 = 1.0$GHz ; $f_2 = 1.00001414$GHz

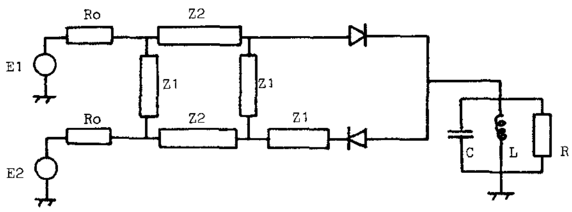

Fig.5 : Balanced two diode mixer
$E1 = 1V\cos(2\Pi f_1 t)$, $f_1 = 2$GHz
$E2 = .1V\cos(2\Pi f_2 t)$, $f_2 = 2.001414$GHz

$R_0 = 50\Omega$ $R = 350\Omega$ $L = .12$mH $C = 100$pF
Z1 : transmission ligne $L = \lambda/4$, $Zc = 50\Omega$
Z2 : transmission ligne $L = \lambda/4$, $Zc = 50/\sqrt{2}\Omega$

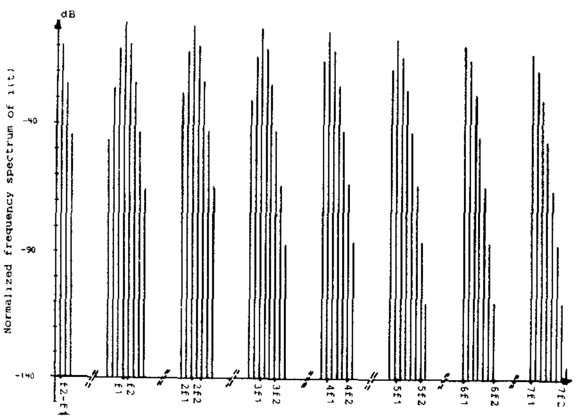

Fig.3 : Normalized frequency spectrum of i(t)
Imax = 9.85 mA

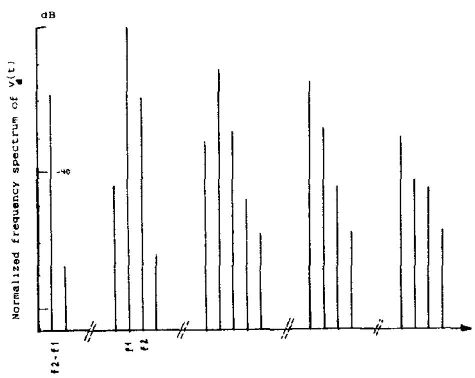

Fig.6 : Spectrum of the voltage vd
Vmax = .503 V